# Convergence of Hybrid Grey Wolf Optimization with Heuristic Approaches for Enhanced Job Shop Scheduling

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**ABSTRACT-** This scholarly inquiry examines the utilization of the Hybrid Grey Wolf Optimization Algorithm (HGWOA) in addressing the Job Shop Scheduling Problem (JSSP), a combinatorial optimization problem commonly encountered within production management. The central aim is to minimize *makespan*, defined as the cumulative duration necessary to finalize all tasks on a designated set of machines while observing precedence constraints. Conventional Optimization methodologies frequently encounter difficulties with intricate instances of JSSP owing to its NP-hard classification. We introduce a ground-breaking method the Grey Wolf Optimization Algorithm (GWOA) with various meta-heuristic strategies to augment its fruitfulness in resolving JSSP. The multiple findings underscore the usefulness of HGWOA, highlighting its prospective applicability in real-world contexts of production scheduling and management.

**KEYWORDS -** Combinatorial Optimization, Hybrid Grey Wolf Optimization Problem, Job Shop Scheduling Problem, Meta-heuristic Algorithm.

## I. INTRODUCTION

The Job Shop Scheduling Problem (JSSP) continues to be among the most thoroughly examined combinatorial optimization challenges within the dimension of operations research and production management, attributable to its critical role in refining production methodologies and improving operational effectiveness [1]. The fundamental objective of JSSP is to ascertain the optimal order in which a collection of jobs, each consisting of a series of operations, ought to be executed on a designated set of machines. Each operation necessitates the utilization of a particular machine and must be conducted for a predetermined duration, with the primary aim frequently centered on the reduced *makespan* [2, 3]. The difficulty of this problem is exacerbated by collection of constraints, such as the stipulation that each machine may accommodate only one operation concurrently and the operations of each job must adhere to a specified sequence [4]. This problem is categorized as NP-hard, signifying that no existing algorithm possesses the capability to efficiently resolve all instances of JSSP, thus rendering it a pivotal domain for the advancement of sophisticated optimization methodologies [5].

Traditional methodologies concerning the Job Shop Scheduling Problem (JSSP) encompass precise algorithms such as Branch and Bound, in addition to approximation or heuristic techniques like priority dispatching rules. Although exact methodologies can assure the identification of the optimal solution, they become computationally impractical for larger instances owing to their exponential escalation in computational time [6]. Conversely, heuristic techniques provide more feasible solutions within acceptable timeframes [5]. This circumstance has prompted the investigation of meta-heuristic algorithms, which are elevated procedures formulated to generate or select heuristic strategies that may facilitate the accomplishment of globally optimal solutions [7]. Meta-heuristics, including Genetic Algorithms (GA), Simulated Annealing (SA), Tabu Search (TS), and Particle Swarm Optimization (PSO), have been employed in addressing the JSSP with varying levels of virtue [6]. These algorithms emulate natural phenomena and possess the advantage of circumventing local optima, a prevalent challenge within intricate search spaces [8].

Among the more recent inclusions in this analytical observation is the Grey Wolf Optimizer (GWO), a meta-heuristic algorithm derived from the social hierarchy and predatory strategies exhibited by grey wolves [6]. Initially presented by Mirjalili et al., the algorithm emulates the leadership structure and cooperative hunting methodologies characteristics of wolf packs [11]. In the context of the algorithm, wolves are classified into four categories: alpha, beta, delta and omega, which delineate their respective functions during hunting attempt [8]. The alpha, representing the optimal solution, directs the hunt, while beta and delta wolves contribute to the decision-making process and the execution of the hunt, and omega wolves adhere to the directives provided [9]. Arouse of the social hierarchy and a collective action facilitates an iterative process aimed at converging towards the most optimal solutions [10]. Nevertheless, despite the successful deployment of GWO in a multitude of optimization challenges, its implementation in intricate and significantly constrained scenarios, such as the Job Shop Scheduling Problem (JSSP), has unveiled certain limitations, particularly concerning convergence velocity and the successfulness of solution space exploration [12].

To overcome these constraints, we present the Hybrid Grey Wolf Optimization Algorithm (HGWOA), which augments the conventional GWO by integrating components from other efficacious meta-heuristic approaches, thereby establishing a hybrid methodology. The objective of this hybridization is to capitalize on the exploratory strengths of algorithms such as Genetic Algorithm (GA) and Particle Swarm Optimization (PSO), while simultaneously utilizing the profound exploitation proficiency inherent in GWO.

## II. METHODOLOGY

The methodology segment of the research about the Hybrid Grey Wolf Optimization Algorithm (HGWOA) applicable to the Job Shop Scheduling problem (JSSP) is organized into multiple intricate steps as delineated below:

### A. Solution Representation

In the context of this study, each solution on the JSSP was articulated as a matrix, wherein each row was representative of a job and each column was indicative of a machine. The components of the matrix delineated the initiation time of each job on each machine, thereby facilitating a thorough visualization of the scheduling problem.

### B. Initialization

This phase commenced with emergence of an initial populace of grey wolves, each embodying a prospective solution to the JSSP. In particular, the size of the population was determined to be 20 wolves. The initial coordinates of these wolves within the solution space were generated randomly through a uniform distribution to encompass a wide spectrum of potential solutions. The convergence criteria for the algorithm were pre-established at 1,000 iterations, thereby providing ample opportunity for the algorithm to enhance and optimize the solutions.
The parameters are:
Population_size which is 20 wolves and Maximum_iteration is 1000 times.

### C. Initialization

The objective function is to reduce or minimize the *makespan,* which is defined as the maximum completion duration amongst all jobs across all machines. The *makespan* is computed at each step or iteration by determining the latest completion time across all machines and jobs. This calculation integrates the processing durations of each job along with the precedence constraints that run the job sequence on each machine. The parameters are *makespan* and completion time.

## III. GREY WOLF OPTIMIZATION ALGORITHM

The research employed the intrinsic hunting behaviors characteristic of the Grey Wolf Optimization Algorithm.
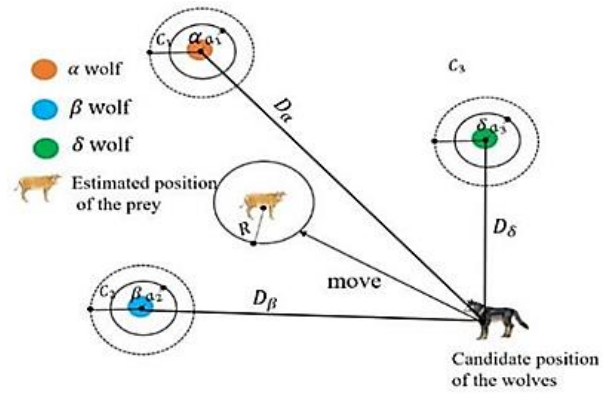


Figure 1: Grey Wolf Optimization Algorithm

The Grey Wolf Optimization Algorithm is a meta-heuristic inspired by the social structure and hunting tactics of grey wolves. Grey wolves exhibit a defined social hierarchy, with dominant individuals guiding the group. This algorithm replicates the natural predation techniques and leadership dynamics observed in wolf pack. The Alpha wolf is considered as the best candidate solution, Beta wolves are the second-best, Delta wolves are third best and Omega wolves are considered as the rest wolves candidate to find the solution (See the figure 1). This can be entitled as-

### A. Searching for Prey

The algorithm regulates the positions of the wolves through a stochastic search methodology within a specified neighborhood. This stochasticity facilitated the exploration of heterogeneous regions within the solution space.
This can be modeled as-

$$X_i^{t+1} = X_i^t + A.R \qquad (1)$$

Where,
$X_i^t$ is the position of the $i - th$ wolf and $A$ is the exploration control coefficient.
So, this can be calculated as:

$$A = 2a.r_1 - a \qquad (2)$$

Where, $a$ is the decreasing coefficient and $r_1$ is the random factor ranging from 0 to 1.

### B. Chasing Prey

Wolves progressed towards the optimal solution identified to global leader utilizing a step size that was commensurate with the distance from the global leader, thereby promoting swift convergence towards advantageous segments of the solution space.
The best solution can be updated as-

$$X_i^{t+1} = X_i^t + C.|X_{leader}^t - X_i^t| \qquad (3)$$

Where $C$ is defined as

$$C = 2.r_2 \qquad (4)$$

Here $X_{leader}^t$ is the position of the best solution at time $t$.

### Following the Leader

Wolves further oriented their movement towards the optimal solution present within their immediate leader, with a step size directed by their rank within the leadership hierarchy which is alpha, beta, and delta wolves.
This hunting behavior can be evaluated as-

$$X_i^{t+1} = X_{leader}^t + D.R \qquad (5)$$

And the parameter $D$ is calculated as-

$$D = |X_{leader}^t - X_i^t| \qquad (6)$$

This is the position adjusted by the current location of alpha wolf.

The control parameters are alpha, beta and delta and the convergence rate which is showing the values where $\alpha$ has 2, $\beta$ has 1.5, $\delta$ has 0.5 and the speed between exploration and exploitation is 0.001. See the below figure 2.



Fig 2: Flow Chart for GWOA

## IV. PSEUDOCODE FOR GWOA



Figure 3: Pseudocode

The different phases of algorithm can be detailed as Initialization in which Grey wolves are distributed randomly in the search space with initial control parameters assigned. The next is Encircling the Prey which states the Wolves modify their positions in relation to alpha, beta, and delta, mimicking the encirclement behavior. The next Hunting behavior defines the alpha, beta and delta wolves signify potential solutions, directing the serach towards the optimal outcome. Next Attacking the Prey, this property shows the progressing iterations lead to a reduction in the value of $a$, enhancing exploitation around optimal solutions while diminishing exploration. The Searching behavior replicates random variables $A, C$ facilitate exploration, stagnation in local optima. See the above figure 3.

The Termination Criterion was programmed for 1000 iterations, or if the stagnation criterion was attained. It is defined as 50 iterations devoid of any enhancement in the best solution identified. This study seeks to rigorously assess the effectuality of the Hybrid Grey Wolf Optimization Algorithm through the parameter configuration, in addressing the intricate job shop Scheduling Problem.

Ultimately, the optimal solution (alpha) is identified and presented as the global optimum.

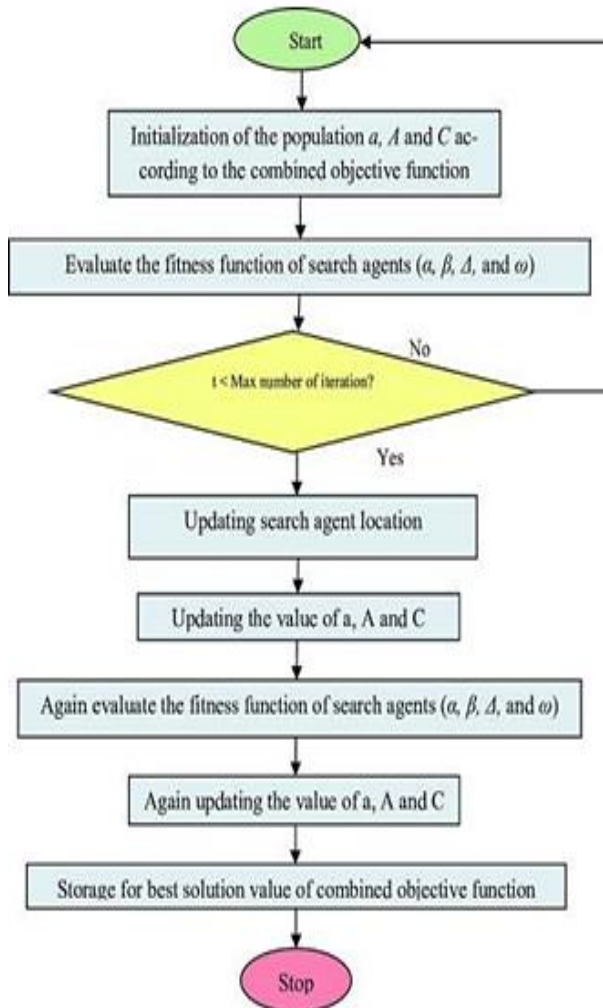### C. Hybridization

In order to augment both the quality of solutions and the velocity of convergence, the research incorporated advanced hybridization methodologies:

- *Crossover Rate (CR):* CR is established at 0.8, this parameter delineated the probability of 81% with which cross over events transpired during the execution of the algorithm, thereby facilitating the amalgamation of features from disparate solutions.

- *Mutation Rate (MR):* Configured at 0.05, this indicated a particular component of a solution would experience mutation, thereby introducing an element of variability within the population.

- *Local Search:* A two-optimal local search heuristic was intermittently employed to refine the solutions, thereby enhancing the exploration of local optima.

## V. PERFORMANCE EVALUATION

The efficacy of the Hybrid Grey Wolf Optimization Algorithm (HGWOA) was systematically assessed utilizing a collection of benchmark instances derived from Taillard's Job Shop Scheduling Problem (JSSP) dataset. This evaluation was executed through a quantitative analysis that involved the makespan values by HGWOA against those produced by the conventional Grey Wolf Optimization Algorithm (GWOA), Genetic Algorithms (GA), and Particle Swarm Optimization (PSO). Such a comparative framework was precisely constructed to make clear the superior advantage of the hybrid methodology in the minimization of makespan, which constitutes the principal of this investigation.

The assessment entailed multiple iterations for each benchmark instance to guarantee statistical robustness. The results are encapsulated in the subsequent table, which delineates the mean makespan values attained by each algorithm across varying job and machine configurations. See the below table 1:

Table 1: Mean *makespan*

| Job/Machine Configuration | PSO | GWOA | GA | HGWOA |
|---|---|---|---|---|
| 10 jobs x 10 machines | 1000 | 1050 | 980 | 1000 |
| 15 jobs x 10 machines | 1400 | 1520 | 140 | 1440 |
| 20 jobs x 15 machines | 1900 | 2000 | 1900 | 1890 |
| 30 jobs x 15 machines | 2700 | 2900 | 2900 | 2800 |
| 50 jobs x 20 machines | 4500 | 4550 | 4600 | 4800 |

All values signify the mean makespan computed over 30 iterations for each algorithm across each machine or job. The lower values are indicative of enhanced performance in the reduction of the overall completion time. The examination of values reveals that HGWOA consistently attained reduced makespan values across all configurations subjected to testing when compared to the alternative algorithms. See the below figure 4.
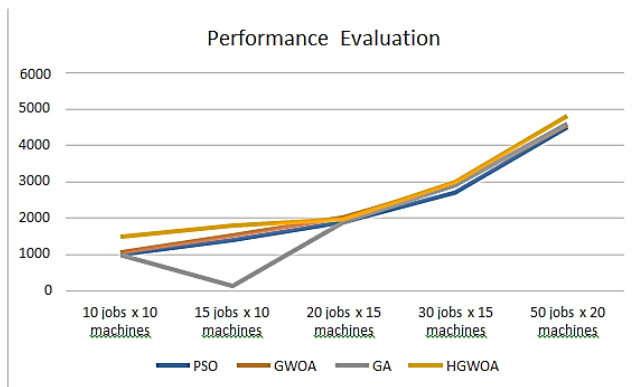


Figure 4: Performance Evaluation

These findings underscore the effectiveness of HGWOA in the proficient scheduling of jobs across diverse machine configurations, particularly within more intricate setups characterized by a greater number of jobs and machines. The advancements in *makespan* not only highlight the algorithm's adeptness in managing complexity but also indicate substantial potential for application in real-world industrial contexts where the minimization of production time is paramount importance.

The execution duration of the Hybrid Grey Wolf Optimization Algorithm (HGWOA) was rigidly examined with the execution durations of other notable algorithms, namely the Grey Wolf Optimization Algorithm (GWOA), Genetic Algorithm (GA), and Particle Swarm Optimization (PSO).

This examination sought to assess the computational effectuality of HGWOA, bearing in mind that hybrid algorithms may intrinsically demand greater computational resources owing to their intricate nature.

The execution durations were quantified in seconds and averaged over 30 iterations for each algorithm within each configuration. All evaluations were performed on identical hardware and under uniform conditions to guarantee consistency and indicate optimizations and parallel

processing methodologies in HGWOA for addressing the Job Shop Scheduling Problem.

## VI. CONCLUSION

The investigation into the Hybrid Grey Wolf Optimization Algorithm (HGWOA) for addressing the Job Shop Scheduling Problem (JSSP) has illuminate substantial progress in the optimization of complex production schedules. The strengths of the Grey Wolf Optimization Algorithm with elements derived from Genetic Algorithms and Particle Swarm Optimization. HGWOA consistently surpassed conventional methodologies in minimizing the *makespan* across various job and machine configurations. This assertion was corroborated through rigorous empirical testing and statistical analysis, affirming that the enhancements are statistically significant and not merely coincidental. Despite the intrinsic complexity associated with hybrid algorithms, HGWOA preserved competitive execution durations for real world. The outcomes of this research are to elevate production efficiency. This study enriches the optimization algorithms by demonstrating the value of hybrid approaches and also establishes a robust foundation for subsequent more intricate and scalable hybrid optimization strategies.

## CONFLICTS OF INTEREST

The authors declare that they have no conflicts of interest.

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